Paper 3C/4C: Further Mechanics 1 Mark Scheme

Question	Scheme	Marks	AOs
1	Use Impulse-momentum principle	M1	2.1
	$2\mathbf{i}-\mathbf{j}=0.5\mathbf{v}-0.5(4\mathbf{i}+\mathbf{j})$	A1	1.1b
	$\frac{1}{2}\mathbf{v}=4\mathbf{i}-\frac{1}{2}\mathbf{j}$, $\mathbf{v}=8\mathbf{i}-\mathbf{j}$ (m s^{-1})	A1	1.1b
	Use of KE $=\frac{1}{2}m\lvert\mathbf{v}\rvert^2-\frac{1}{2}m\lvert\mathbf{u}\rvert^2$	M1	2.1
	$=\frac{1}{2}\times 0.5\times\{(64+1)-(16+1)\}$	A1	1.1b
	$=\frac{1}{4}\times 48=12$ (J) *	A1*	1.1b
		(6)	
			(6 marks)

Notes:

M1: Difference of terms & dimensionally correct
A1: Correct unsimplified equation
A1: cao
M1: Must be a difference of two terms
Must be dimensionally correct
A1: Correct unsimplified equation
A1*: Complete justification of given answer

Question	Scheme	Marks	AOs
2(a)	$R = 5g\cos\alpha\left(=5g\times\frac{4\sqrt{3}}{7}=48.497...\right)$	M1	3.4
	Force due to friction = $\mu\times 5g\cos\alpha$	M1	3.4
	Work-Energy equation	M1	3.4
	$\frac{1}{2}\times 5\times 64 = 5\times 9.8\times 14\sin\alpha + 14\mu R$	A1	1.1b
	$\mu = 0.0913$ or 0.091	A1	1.1b
		(5)	
(b)	Appropriate refinement	B1	3.5c
		(1)	
		(6 marks)	

Notes:

(a)

M1: Condone sin/cos confusion

M1: Use of $\mu\times$ their R

M1: Must be using work-energy. Requires all terms
Condone sin/cos confusion, sign errors and their R

A1: Correct in θ and μR

A1: Accept 0.0913 **or** 0.091

(b)

B1: e.g.
- do not model the parcel as a particle and therefore take air resistance into account
- take into account the dimensions/uniformity of the parcel

Question	Scheme	Marks	AOs
3(a)	Use NEL to find the speed of particle after the first impact $= eu = \frac{3}{4}u\frac{\pi}{2}$	B1	3.4
	Impulse = $\lambda mu = mv - mu = \pm\left[\frac{3}{4}mu - (-mu)\right]$	M1	3.1b
	$\lambda = \frac{7}{4}$	A1	1.1b
		(3)	
(b)	Use NEL to find the speed of the particle after the second impact $= \frac{3}{4} \times \frac{3}{4}u = \frac{9}{16}u$	B1	3.4
	Use of $s = vt$ to find total time	M1	3.1b
	$7 = \frac{2}{u} + \frac{4}{\frac{3}{4}u} + \frac{2}{\frac{9}{16}u}\left(= \frac{2}{u} + \frac{16}{3u} + \frac{32}{9u}\right)$	A1	1.1b
	Solve for u: $63u = 18 + 48 + 32$	M1	1.1b
	$u = \frac{98}{63} = \frac{14}{9}\left(= 1.\dot{5}\right)$	A1	1.1b
		(5)	
			(8 marks)

Notes:

(a)
B1: Using Newton's experimental law as a model to find the speed after the first impact
M1: Must be a difference of two terms, taking account of the change in direction of motion
A1: cao

(b)
B1: Using NEL as a model to find the speed after the second impact
M1: Needs to be used for at least one stage of the journey
A1: Ur equivalent
M1: Solve their linear equation for u
A1: Accept 1.56 or better

Question	Scheme	Marks	AOs
4(a)	Complete strategy to find the kinetic energy after the second impact	M1	3.1b
	Parallel to AB after collision: $u\cos 60°$	M1	3.1b
	Perpendicular to AB after collision: $\frac{1}{\sqrt{3}}u\sin 60°$	M1	3.4
	Components of velocity after first impact: $\frac{u}{2}, \frac{u}{2}$	A1	1.1b
	Parallel to BC after collision: $\frac{u}{2}$ $\left(u\times\frac{1}{\sqrt{3}}\sin 60°\right)$	M1	3.1b
	Perpendicular to BC after collision: $\sqrt{\frac{2}{5}}\times\frac{u}{2}\left(=\frac{1}{\sqrt{10}}u\right)$ $\left(\sqrt{\frac{2}{5}}\times u\cos 60°\right)$	M1	3.4
	Components of velocity after second impact: $\frac{u}{2}, \frac{u}{\sqrt{10}}$	A1	1.1b
	Final KE $=\frac{1}{2}m\left(\frac{u^2}{4}+\frac{u^2}{10}\right)$ $\left(=\frac{mu^2}{2}\times\frac{7}{20}\right)$		
	Fraction of initial KE $=\dfrac{\frac{mu^2}{2}\times\frac{7}{20}}{\frac{mu^2}{2}}=\frac{7}{20}=35\%$ *	A1*	2.2a
		(8)	
(b)	The answer is too large - rough surface means resistance so final speed will be lower	B1	3.5a
		(1)	
		(9 marks)	

Notes:

(a)

M1: Use of CLM parallel to the wall. Condone sin/cos confusion

M1: Use NEL as a model to find the speed perpendicular to the wall. Condone sin/cos confusion

A1: Both components correct with trig substituted (seen or implied)

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M1: Use NEL as a model to find the speed perpendicular to the wall. Condone sin/cos confusion

A1: Both components correct with trig substituted (seen or implied)

M1: Correct expression for total KE using their components after 2nd collision

A1*: Obtain **given answer** with sufficient working to justify it

(b)

B1: Clear explanation of how the modelling assumption has affected the outcome

Question	Scheme	Marks	AOs
5(a)	Use of $P = Fv$: $F = \dfrac{12000}{20}$	B1	3.3
	Equation of motion: $F - (200 + 2v) = 600a$	M1	3.4
	$600 - 240 = 600a$	A1ft	1.1b
	$360 = 600a$, $a = 0.6$ (m s^{-2})	A1	1.1b
		(4)	
(b)	Equation of motion:	M1	3.3
	$\dfrac{12000}{w} - (200 + 2w) - 600g\sin\theta = -600 \times 0.05$	A1 A1	1.1b 1.1b
	3 term quadratic and solve: $2w^2 + 590w - 12000 = 0$	M1	1.1b
	$w = \dfrac{-590 + \sqrt{590^2 + 96000}}{4} = 19.1$ (m s^{-1})	A1	1.1b
		(5)	
			(9 marks)

Notes:

(a)

B1: 600 or equivalent

M1: Use the model to form the equation of motion
Must include all terms .Condone sign errors

A1ft: Correct for their F

A1: cao

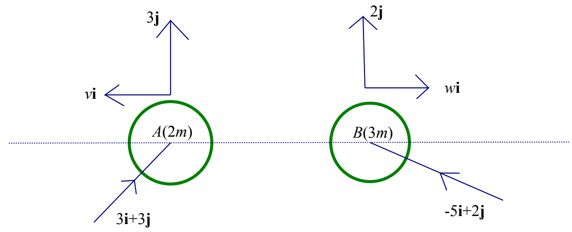
(b)

M1: Use the model to form the equation of motion
All terms needed. Condone sign errors and sin/cos confusion

A1: All correct A1A1
One error A1A0

M1: Dependent on the preceding M1. Use the equation of motion to form a 3-term quadratic in w only

A1: Accept 19. Do not accept more than 3 s.f.

Question	Scheme	Marks	AOs
6(a)			
	Overall strategy to find $\mathbf{v}_A$	M1	3.1a
	Velocity of A perpendicular to loc after collision $= 3\mathbf{j}$ (m s^{-1})	B1	3.4
	CLM parallel to loc	M1	3.1a
	$2m \times 3 - 3m \times 5 = 3mw - 2mv \quad (-9 = 3w\text{-}2v)$	A1	1.1b
	Correct use of impact law	M1	3.1a
	$v + w = \frac{1}{4}(3+5) \quad (=2)$	A1	1.1b
	Solve for w $\quad 3w - 2v = -9$ $2v + 2w = 4$		
	$\mathbf{v}_B = -\mathbf{i} + 2\mathbf{j}$ (m s^{-1}),	A1ft	1.1b
		(7)	
(b)	$\cos\theta = \frac{(-5\mathbf{i}+2\mathbf{j}).(-\mathbf{i}+2\mathbf{j})}{\sqrt{29}\sqrt{5}}$	M1	3.1a
	$\theta = 41.63...° = 42°$ (nearest degree)	A1	1.1b
	Alternative method: $\tan^{-1} 2 - \tan^{-1}\frac{2}{5} = 41.63...° = 42°$ (nearest degree)		
		(2)	
			(9 marks)

Notes:

(a)

M1: Correct overall strategy to form sufficient equations and solve for $\mathbf{v}_A$

B1: Use the model to find the component of $\mathbf{v}_A$ perpendicular to the line of centres

M1: Use CLM to form equation in v and w. Need all 4 terms, dimensionally correct

A1: Correct unsimplified

M1: Must be used the right way round

A1: Correct unsimplified

A1ft: $\mathbf{v}_B$ correct. Follow their $2\mathbf{j}$

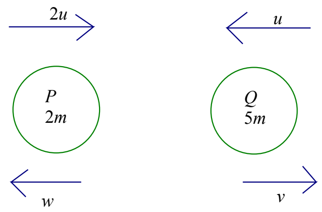
(b)

M1: Complete method for finding the required angle. Follow their $\mathbf{v}_B$

A1: cao

Question	Scheme	Marks	AOs
7(a)	In equilibrium $\Rightarrow$ no resultant vertical force	M1	2.1
	$\frac{3mgx}{a} = mg$	A1	1.1b
	$x = \frac{a}{3}$, $d = \frac{4}{3}a$ *	A1*	2.2a
		(3)	
(b)	Equation of motion:	M1	3.1a
	$\frac{3mga}{a} - mg = m\ddot{x}$	A1	1.1b
	$\ddot{x} = 2g$	A1	1.1b
		(3)	
(c)	Max speed at equilibrium position	B1	3.1a
	Work energy & use of EPE $= \frac{\lambda x^2}{2a}$	M1	3.1a
	$\frac{3mga^2}{2a} = \frac{3mg\left(\frac{a}{3}\right)^2}{2a} + \frac{1}{2}mv^2 + mg\frac{2a}{3}$	A1 A1	1.1b 1.1b
	$\frac{1}{2}v^2 = ga\left(\frac{3}{2} - \frac{1}{6} - \frac{2}{3}\right) = \frac{2}{3}ga$, $v = \sqrt{\frac{4ga}{3}}$	A1	1.1b
		(5)	
(d)	At max ht. KE = 0. EPE lost = GPE gained	M1	3.1a
	$\frac{3mga^2}{2a} = mgh$	A1	1.1b
	$OB = \frac{a}{2}$	A1	1.1b
		(3)	
			(14 marks)

Question 7 notes:
(a) **M1:** Use $T = \frac{\lambda x}{a}$ to form equation for equilibrium **A1:** Correct unsimplified equation **A1*:** Requires sufficient working to justify given answer plus a 'statement' that the required result has been achieved
(b) **M1:** Use $T = \frac{\lambda x}{a}$ to form equation of motion Need all 3 terms. Condone sign errors **A1:** Correct unsimplified equation **A1:** cao
(c) **B1:** Seen or implied **M1:** Form work-energy equation. All 4 terms needed Condone sign errors **A1:** Correct unsimplified equation A1A1 One error in the equation A1A0 **A1:** cao
(d) **M1:** Form energy equation **A1:** Correct unsimplified equation **A1:** cao

Question	Scheme	Marks	AOs
8(a)			
	Complete overall strategy to find v	M1	3.1a
	Use of CLM	M1	3.1a
	$2m\times 2u-5m\times u=5m\times v-2m\times w$, $(-u=5v-2w)$	A1	1.1b
	Use of Impact law:	M1	3.1a
	$v+w=e(2u+u)$	A1	1.1b
	Solve for v: $-u=5v-2w$ $6eu=2v+2w$		
	$7v=u(6e-1)$ $\left(v=\frac{u}{7}(6e-1)\right)$	A1	1.1b
	Direction of Q reversed: $v>0$	M1	3.4
	$\Rightarrow 1\geq e>\frac{1}{6}$	A1	1.1b
		(8)	
(b)	$e=\frac{1}{3}\Rightarrow v=\frac{u}{7}$, $w=\frac{6u}{7}$	B1	2.1
	Equation for KE lost	M1	2.1
	$\frac{1}{2}\times 2m\left(4u^2-\frac{36u^2}{49}\right)+\frac{1}{2}\times 5m\left(u^2-\frac{u^2}{49}\right)$	A1 A1	1.1b 1.1b
	$\frac{1}{2}mu^2\left(8-\frac{72}{49}+5-\frac{5}{49}\right)=\frac{40mu^2}{7}$ *	A1*	2.2a
		(5)	
(c)	Increase $e\Rightarrow$ more elastic $\Rightarrow$less energy lost	B1	2.2a
		(1)	
			(14 marks)

Question 8 notes:	
(a)	
M1:	Complete strategy to form sufficient equations in v and w and solve for v
M1:	Use CLM to form equation in v and w
	Needs all 4 terms & dimensionally correct
A1:	Correct unsimplified equation
M1:	Use NEL as a model to form a second equation in v and w. Must be used the right way round
A1:	Correct unsimplified equation
A1:	for v or $7v$ correct
M1:	Use the model to form a correct inequality for their v
A1:	Both limits required
(b)	
B1:	Or equivalent statements
M1:	Terms of correct structure combined correctly
A1:	Fully correct unsimplified A1A1
	One error on unsimplified expression A1A0
A1*:	cso. plus a 'statement' that the required result has been achieved
(c)	
B1:	"less energy lost" or equivalent